General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2B

MM2B

ASSESSMENT and QUALIFICATIONS ALLIANCE

Monday 16 January 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A stone, of mass 0.4 kg, is thrown vertically upwards with a speed of $8\,\mathrm{m\,s^{-1}}$ from a point at a height of 6 metres above ground level.

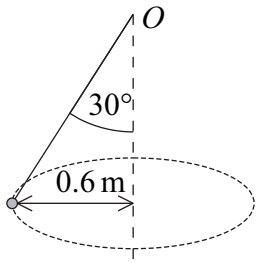
(a) Calculate the initial kinetic energy of the stone. *(2 marks)*

(b) (i) Show that the kinetic energy of the stone when it hits the ground is 36.3 J, correct to three significant figures. *(2 marks)*

(ii) Hence find the speed at which the stone hits the ground. *(3 marks)*

(iii) State one assumption that you have made. *(1 mark)*

2 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O. The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O.



(a) Show that the tension in the string is 22.6 N, correct to three significant figures. *(3 marks)*

(b) Find the speed of the particle. *(4 marks)*

3 A particle moves in a straight line and at time t has velocity v, where

$$v = 2t - 12e^{-t}, \quad t \geqslant 0$$

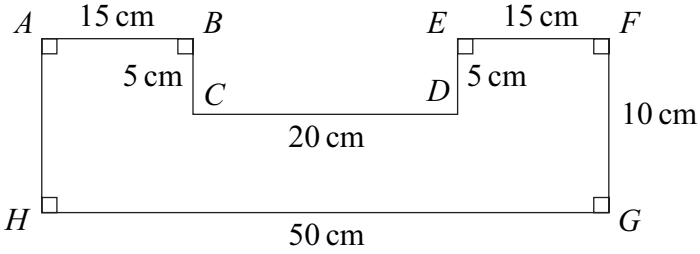
(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) State the range of values of the acceleration of the particle. *(3 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

4 The diagram shows a uniform lamina $ABCDEFGH$.



(a) Explain why the centre of mass is 25 cm from AH. *(1 mark)*

(b) Show that the centre of mass is 4.375 cm from HG. *(4 marks)*

(c) The lamina is freely suspended from A. Find the angle between AB and the vertical when the lamina is in equilibrium. *(4 marks)*

(d) Explain, briefly, how you have used the fact that the lamina is uniform. *(1 mark)*

5 A particle moves such that at time t seconds its acceleration is given by

$$(2\cos t\mathbf{i} - 5\sin t\mathbf{j})\,\mathrm{m\,s^{-2}}$$

(a) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 0$. *(3 marks)*

(b) When $t = 0$, the velocity of the particle is $(2\mathbf{i} + 10\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find an expression for the velocity of the particle at time t. *(5 marks)*

6 A student is modelling the motion of a small boat as it moves on a lake. When the speed of the boat is $12\,\text{m s}^{-1}$, the engine is switched off. At time t seconds later, it has a velocity of $v\,\text{m s}^{-1}$ and experiences a resistance force of magnitude $20v$ newtons. The mass of the boat is 80 kg.

To set up a simple model for the motion of the boat, the student assumes that the water in the lake is still and that the boat travels in a straight line.

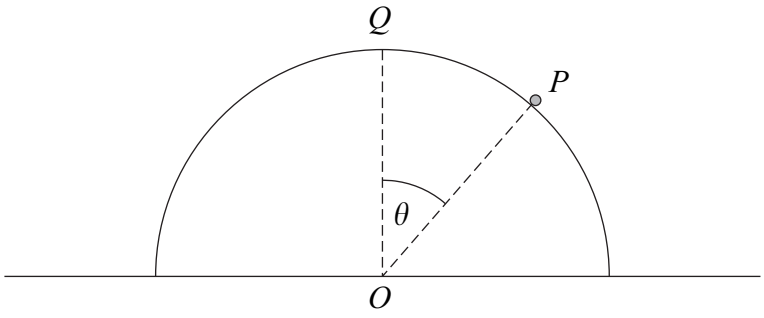
(a) Explain how these two assumptions allow the student to create a simple model. *(2 marks)*

(b) State one other assumption that the student should make. *(1 mark)*

(c) (i) Express $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ in terms of v. *(2 marks)*

(ii) Find an expression for v in terms of t. *(5 marks)*

7 A particle P, of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O. The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of $2\,\text{m s}^{-1}$. When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v\,\text{m s}^{-1}$.



(a) Show that $v^2 = 4 + 6g(1 - \cos\theta)$. *(4 marks)*

(b) Find the value of θ when the particle leaves the hemisphere. *(5 marks)*

8 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point O.

(a) Find the length of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(b) The particle is pulled down and held at a point P, which is 1 metre vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 45 J. *(2 marks)*

(c) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\mathrm{m\,s^{-1}}$ when it is x metres **below O**.

(i) Show that, while the string is taut,

$$v^2 = 39.6x - 25x^2 - 14.6$$ *(7 marks)*

(ii) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(3 marks)*

END OF QUESTIONS

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